A New Model for Strategy Development

Combining Categorical Data Analysis with Growth Modeling

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Abstract

Latent Growth Modeling (LGM) is a widely applied, powerful and flexible statistical method used to analyze longitudinal data (Bollen & Curran, 2006; Duncan, Duncan & Strycker, 2006). While it has several advantages in analyzing longitudinal data, it is less suited to study strategy development as it is focused on inter-individual variation of the variable of interest at specific points in time and not on inter-individual variation in terms of development across time. The Overlapping Waves Model, introduced by Siegler (1996), is a metaphor used to illustrate a typical sequence of increasing and decreasing use of cognitive strategies during development. Although it overcomes the limitation of LGM, at least at the conceptual level, it fails to be useful when the individuals do not start to use the strategies at the same time. To tackle this problem, a new model synthesized from Item Response Theory (IRT) and LGM is proposed. IRT provides the means to relate the use of strategies to an underlying developmental dimension. Movement of individuals along this dimension can be modeled by means of LGM. Visualization (3D) is used to illustrate. Examples of successful application of the model to microgenetic studies (using commercially available software) are presented.

Keywords: Latent Growth Modeling, strategy development, Overlapping Waves Model, Item Response Theory

A New Model for Strategy Development: Combining Categorical Data Analysis with Growth Modeling

Latent Growth Modeling (LGM) is a widely applied, powerful and flexible statistical method used to analyze longitudinal data (Bollen & Curran, 2006; Duncan et. al., 2006). Figure 1 is an example of raw longitudinal data for several individuals. LGM has several advantages for analyzing such longitudinal data, as will be outlined below.

However, when analyzing development in the use of various strategies, depictions as in Figure 1 do not apply easily. Suppose we have several possible ways of solving a problem, like e.g. simple, intermediate, and advanced (in terms of difficulty). Less developed participants will most likely use simple strategies (or be able to solve simple tasks only). More developed participants will most likely use intermediate strategies (or be able to solve more challenging tasks), but occasionally use the simple strategy also, and perhaps even seem to use the more advanced strategies too. The most developed participants are likely to use predominantly, but certainly not exclusively, the most advanced strategies (and be able to solve the most difficult tasks).

This state of affairs is better captured in Siegler's Overlapping Waves Model (OWM, see Figure 2). It depicts development as a sequence of overlapping waves and can be used conceptualize the individual latent developmental dimension as movement along the x axis.

However, even this model fails when there are differences in the individual participants in when they start using a new strategy. And of course there are, and depending on the topic we study these might be small, but more often they are huge. Unfortunately, variability in starting point along the developmental dimension makes averaging impossible which consequently disables the usage of regression and growth modeling.

To overcome this problem, we need to be able to map variation at fixed points in time to real developmental variation in terms of being closer or further away from milestones along the developmental dimension.

This paper first reviews the role of LGM in analyzing longitudinal data and discusses its advantages and limitations. Second, it will frame OWM as an instance of a multi-category Item Response Theory (IRT) model. Third, it will propose a new model synthesized from LGM and Item Response Theory (IRT) to overcome the limitation of LGM in studying strategy development. Finally, it provides examples of how the new proposed model can be applied to microgenetic studies.

Latent Growth Modeling (LGM)

Latent variable Growth curve Modeling (LGM) is a powerful and flexible technique, which can be used to model longitudinal development (Bollen & Curran, 2006; Duncan, Duncan & Strycker, 2006). Linear LGM presupposes a steady increase or decrease in the target variable over a small number of measurement occasions equally spaced for each person. The increase is assumed to be linear, with an intercept and a slope parameter describing a trajectory for each individual. These intercepts and slopes are assumed to be different for each respondent and normally distributed in the population with unknown mean and variance. In the usual case, the observed scores for an individual participant depart from his or her best fitting straight line, and it is assumed that these residuals are normally distributed in the population with zero mean and certain variance for each measurement. Moreover, these residuals are not correlated between measurements. A linear LGM seems well suited to model increasing maturity over measurement occasions and, if the model holds, every participant's trajectory can be represented by a straight line. But the model can also be extended with e.g. a quadratic growth parameter to model curved trajectories. Growth parameters (intercept and slope) can be related to predictors and dependents. An LGM can also be used for short time longitudinal studies of e.g. five weeks long, known as microgenetic studies.

Limitations of LGM

Although LGM is very well suited for longitudinal data analysis, it does have its limitations. Whereas an LGM is focused on inter-individual variation at fixed time points, what we really are interested in, is developmental variation. Developmental variation refers to differences along a developmental dimension. For example, if A is more developed than B this may mean that B is perhaps at a level where A was earlier. In studying development in strategy use, in particular in a longitudinal design, such developmental variation is of much more interest than individual variation in terms of the variable of interest at a given time point *t*. Therefore, to overcome the limitation of LGM, variation needs to be turned.

A standard LGM works well for studying development in case of a linear increase, such as length in young children in which later (time) and longer (growth or development) coincide and both kinds variation are the same. However, it does not work well in case of a phenomenon that is coming up and disappearing again (like the use of counting on fingers). In this case, at a certain measurement occasion or a certain age, some participants are at that point in their development, while some have not yet reached this milestone and the remaining participants have already moved beyond using this strategy. In addition, there may be large individual differences. Hence, we need a better model, which brings us to the Overlapping Waves Model (OWM).

In sum: regression techniques including LGM are based on, and require, variation between participants at the measurement points (so at fixed time points), however, this is not useful for studying differences in strategy use along a developmental dimension, since this relates to inter-individual variation in terms of when (at which point in time) the strategies are predominantly used.

Overlapping Waves Model (OWM) and Item Response Theory (IRT)

The Overlapping Waves Model (OWM) was developed by Siegler (1996) to illustrate a typical pattern consisting of a sequence of increasing and decreasing use of ways of thinking (strategies) during development. It was mainly used with cognitive strategies. For example, adding numbers: 2+7 = 9, using fingers, count on, count from largest to retrieval. Those strategies are qualitatively different approaches used to solve the problem. Figure 4 is an OWM which illustrates changes in use of different strategies at different ages. This model overcomes the problem if LGM since it depicts true development and can be used to study differences of variables along a developmental dimension.

Limitation of the OWM

For the OWM to work for analyzing strategic development, it would have to be the case that the participants start new strategies at the same time. However, it is more likely that if there are differences between individual participants in when they start (earlier - later) using a new strategy, and in this case averaging becomes impossible (see Figure 5). Consequently, we cannot use growth modeling or regression. Fortunately, this problem can be solved by combining LGM with Item Response Theory (IRT).

Item Response Theory (IRT.)

IRT comprises of analysis techniques developed for categorical data like ordinal data with few categories (non-negative and discrete data; e.g. a score from 1 to 5). It is heavily used by test constructors.

This is helpful because IRT methods can relate strategy use (a qualitative variable) to a hypothetical underlying ability.

Suppose participants are presented with several problems (items), each invoking a choice from a few response possibilities, suppose furthermore —following IRT modeling— that the likelihood of using one of the responses depends on a single latent variable by a mathematical function known as the item response function. The Partial Credit Model is particularly useful to model responses that are ordered as a series of steps that must be mastered in sequence. The latent variable represents inter-individual differences in ability, which in this case translates to being more or less advanced in terms of strategy use. Figure 3 illustrates. Figure 2 in this paper is based on a microgenetic study by van der Ven, Boom, Kroesbergen, & Leseman (2012), just as an example. For now it is enough to know there are 5 strategies involved.

The X-axis of Figure 2 can be thought of as representing strategy maturity differences between participants (as person characteristic). The more to the right on the X-axis, the higher the latent ability and the more likely it becomes that a participant selects relatively more advanced strategies. The Y-axis shows the likelihood of using each strategy, given a latent ability. For example, strategy-5-use strongly increases with increasing maturity, while strategy-1-use diminishes rapidly with increasing maturity.

Alternatively, the X-axis can be used to represent strategy characteristics; e.g. the peak of each of the middle curves coincides with the ability for which that strategy is most likely to be used, although it is also clear there is considerable overlap between strategies.

The result is a latent strategy maturity scale that can be used to represent inter-individual ability differences and strategy advancedness. The position along this strategy scale is nonlinearly and probabilistically related to the use of the various strategies. The attractiveness of the transformation of the categorical scores to this unbounded continuous interval scale is that it opens up the possibility to use all the usual regression techniques, including LGM.

Conclusion IRT part

Using IRT models in this way enables one of the key ideas presented in this paper: the idea that the emerging latent ability as shown on the X-axis can be used to model intra-individual development as in Siegler's Overlapping Waves Model. In other words: development over time can be conceptualized and visualized as a shift to right in Figure 2. The result in terms of expected strategy use can be quite complex to describe because it depends on the starting point and the growth of the particular subject. Only the first and the last strategy have consistent change patterns, the use of all other strategies increases first and decreases later. The exact profile of these shapes depends on item characteristics and may be different for different items. In a Partial Credit model the basic shape (steepness of the curves) is fixed. Although scaling of X-axis is arbitrary, location (placement to the right or to the left for the whole set of curves) can vary by item. Height of the curves, or area's underneath it, which can also be expressed as the distance between crossings of curves, may be different within or between items.

Basic principles of relevant IRT modeling, and some alternative models, are reviewed briefly and very accessible by Millsap (2010); a more elaborate introduction on IRT is given by Ayala (2009); more details on polytomous item response models can be found in Ostini and Nering (2006); and more on categorical data-analysis in general in Agresti (2002).

Three Dimensional Overlapping Waves Model: LGM and IRT combined

The Overlapping Waves model of Figure 1 was originally presented to visualize development, whereas the polytomous item response model of Figure 2 is more commonly used to depict individual differences. With a three-dimensional version of the Overlapping Waves model both uses can be combined.

In Figure 3 the Y-axis refers to individual differences, the X-axis to time (measurement occasions = eight weeks in our empirical example), and the Z-axis refers to probability of using one of five strategies. The floor is a two dimensional plane on which a growth curve model can be placed as illustrated in Figure 7.

In Figure 4 estimated (idealized) individual trajectories of development in strategy maturity are shown on the floor plane for a subsample of 20 participants. For one individual, as illustration, the implied item response functions are depicted in the Z-plane. The degree of curvature is limited, which makes sense, because it represents an increase from -1.7 to 1.7 on the strategy maturity scale as in Figure 3. Figures 3 and 4 are combined in Figure 5 to illustrate that each individual curve from Figure 4 follows the surface of the waves in Figure 3.

The more growth, the more the set of curves for a particular individual turns away from an orientation parallel to the week axis, and the more curvature will result. No growth would result in straight lines for each of the 5 strategies. But a different starting point (different intercept in the growth model part) can also lead to completely different curvatures: imagine the set of curves shifted along the latent ability dimension.

Example: Stepwise Understanding Randomness

The general model has been applied to several data sets now (see e.g. Van der Ven, Boom, Kroesbergen, & Leseman, 2012). The pilot study presented here is just meant as an example, no substantial conclusions should be drawn from this yet (more research is planned)[1].

Children's understanding of randomness was studied using a microgenetic design in three age cohorts (Grade 1, 3, and 5) of different primary schools in rural parts of the Netherlands. During five weeks three probability-related tasks were administered weekly to 77 children. Only data for the spinner tasks will be presented. Inspired by Metz (1998), spinner games were used with two or three colors (unequal surface). Children could choose a color and win or lose. Eight rounds were played. The total score was indicated by a pawn on a score board.

For eight topics questions were asked before, during, or after playing the game: e.g. (with spinner with ¾ red and ¼ yellow): "which color do you want to play with and why?"; "who do you think is going to win and why."; "can you still win?"; and "could you have won with this spinner?". For each topic the answers given by each child were coded using 4 possible categories (based on Metz, 1998). Developmental progress was presumed to go from: (1) No understanding at all (e.g. I like red) to (2) determinism which is denial of chance element (If I turn fast), to (3) unpredictability (you never know), to (4) recognizing some degree of long term predictability (dominant color is best bet). Randomness is not a part of curriculum in the Netherlands. Participants were not given feedback by the test administrators, but were able to see, of course, the outcomes when the game was eventually played each week. This resulted in a 77 by 40 (= 5 x 8) raw data-matrix with codes one to four.

Results

[1] I would like to thank my students: Brake, M., Bruseker, K., Hbaba, G, & van 't Spijker, R. for collecting the data.

Figures 6 shows that the eight items (= topic/question set) had a very different profile. These shapes of these item profiles are fixed over the 5 weeks (to achieve measurement invariance). On the x-axis is the difficulty of the item (to use the IRT parlance) which in this case reflects whether the particular question tends to elicit more advanced or more simplistic responses. Note that children can be (but are not) placed on the same x-axis scale. In that case actual abilities will cover only small part of the scale, centered around zero, because the average is arbitrarily set to zero.

Figures 7 shows the expected changes in the use of the categories (levels of understanding), over weeks, for each item (topic/question set), separately (corresponding to Figure 6).

Figure 8 Shows that the expected and observed means of probability of use of four different developmental progress stages in different weeks are close.

Discussion

A new 3D Overlapping Waves model was presented, based on a combination of Latent variable Growth curve Modeling (LGM) and Item Response Theory (IRT) modeling. The statistical principles that were used are long established and sound. It is a formal model for conceptualizing strategy development which sheds new light on the issue of variability in development. It is also an empirically testable model that might be helpful in longitudinal studies in which the nature of responding does change fundamentally over development or experience.

All advantages of LGM apply to this model. Predictors can be added to the LGM part, it is also possible to test nonlinear growth, or to add more growers to the model. All advantages of IRT modeling also apply. The new part is that what is normally the end result (estimated individual scores or group indicators thereof) is now —in an additional step— transformed in a set of thresholds which can be visualized as a set of curves (with strong shape constraints). The average ability of the participants shifted up along the latent dimension.

The raw data may appear incredibly complex and variable, but, as shown, these complex data may still reflect relatively simple linear growth along an underlying dimension. Of course, in actual practice there may always be violations of the assumptions, for all kinds of reasons, but if there is a consistent pattern over individuals to a sufficient degree, such an underlying dimension is plausible. The analysis has important theoretical implications and can be done with commercially available software and is not difficult to conduct, although it requires some conceptual work and spatial imagination.

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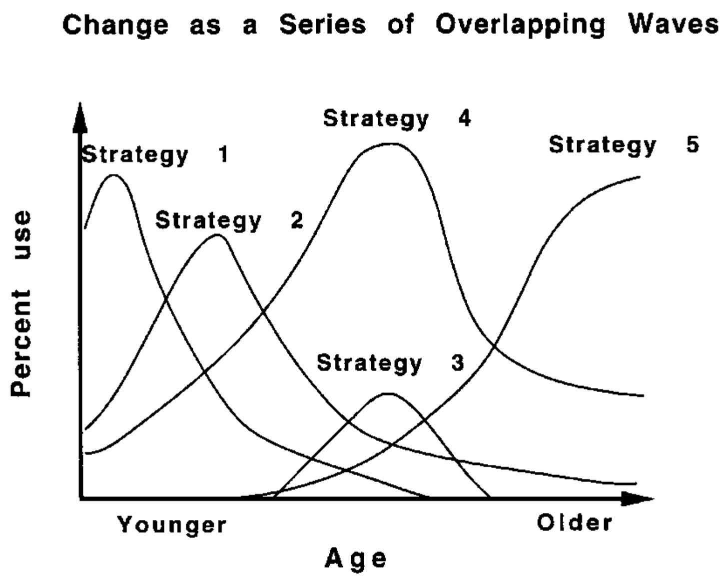

Fig. 1 Overlapping waves depiction of cognitive development. Data from "*Emerging Minds*", by R. Siegler, 1996, New York: Oxford University Press, p. 89.

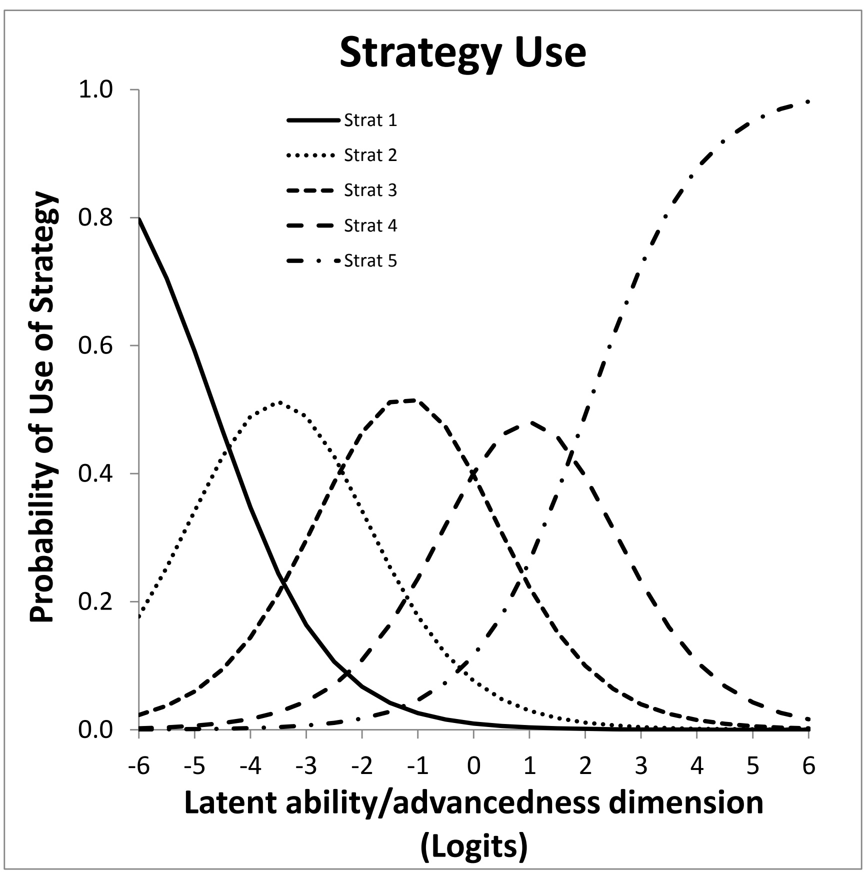

Fig. 2 Response Function for a Five-Category Polytomous Item

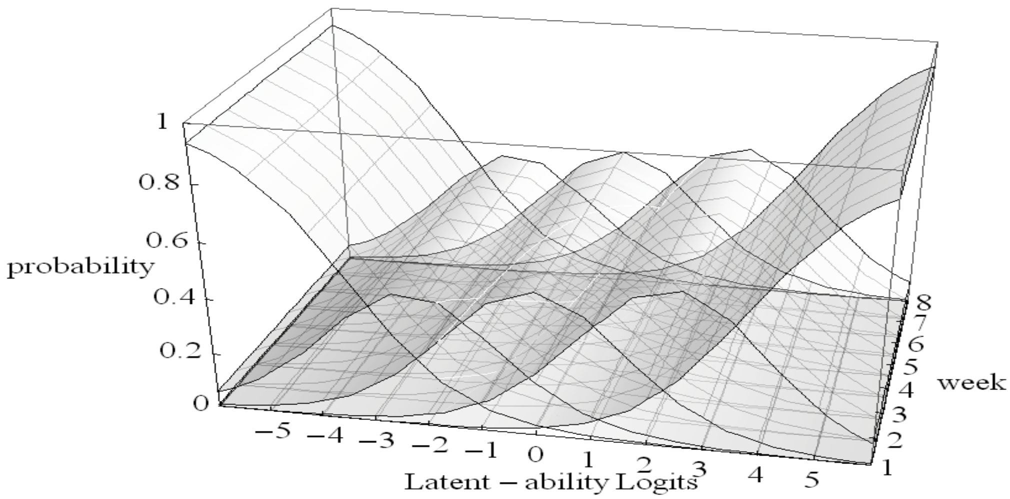

Fig. 3 3D-Overlapping Waves Model. The Y-axis refers to individual differences, the X-axis to time (week number), and the Z-axis to probability of using one of the five strategies.

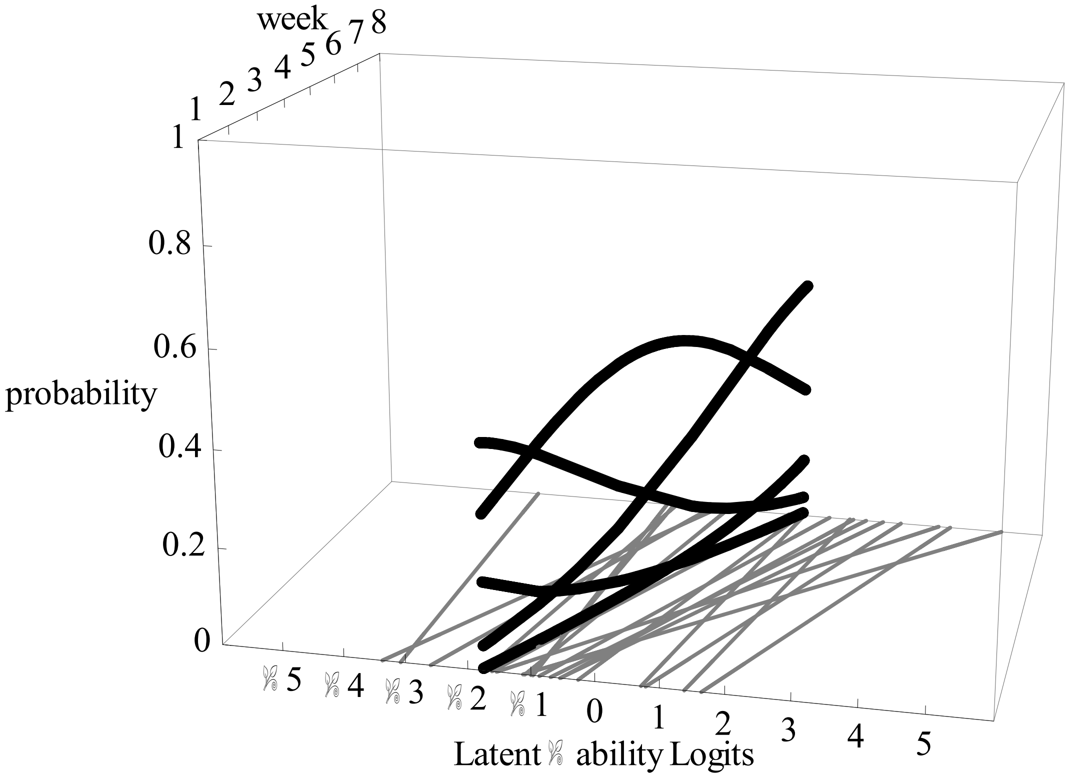

Fig. 4 LGM trajectories (thin grey lines) and snapshot of Response Functions for one participant (heavy black lines; each representing a strategy). See Figure 3 for more details.

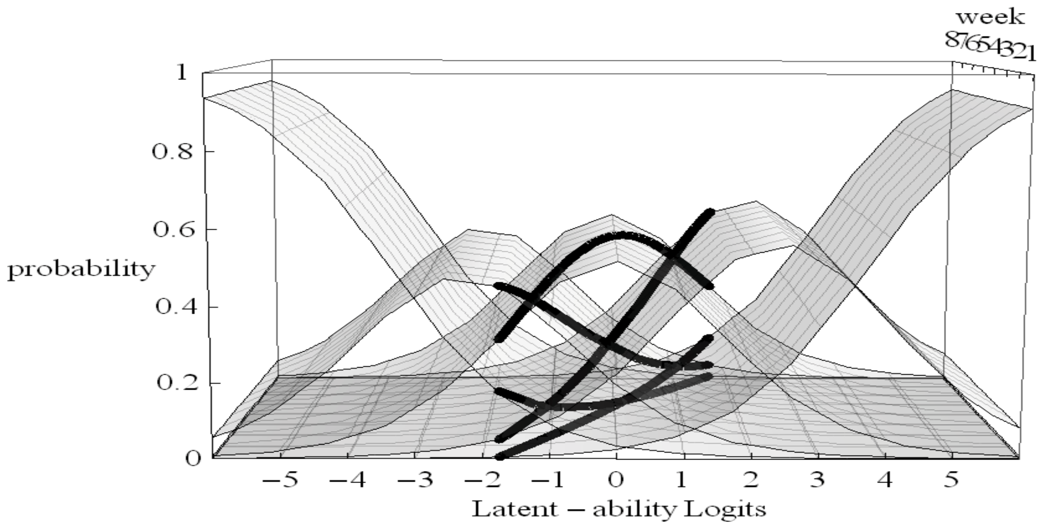

Fig. 5 3D-Overlapping Waves Model with Response Functions for one participant as an illustration. See Figures 7 for more details.

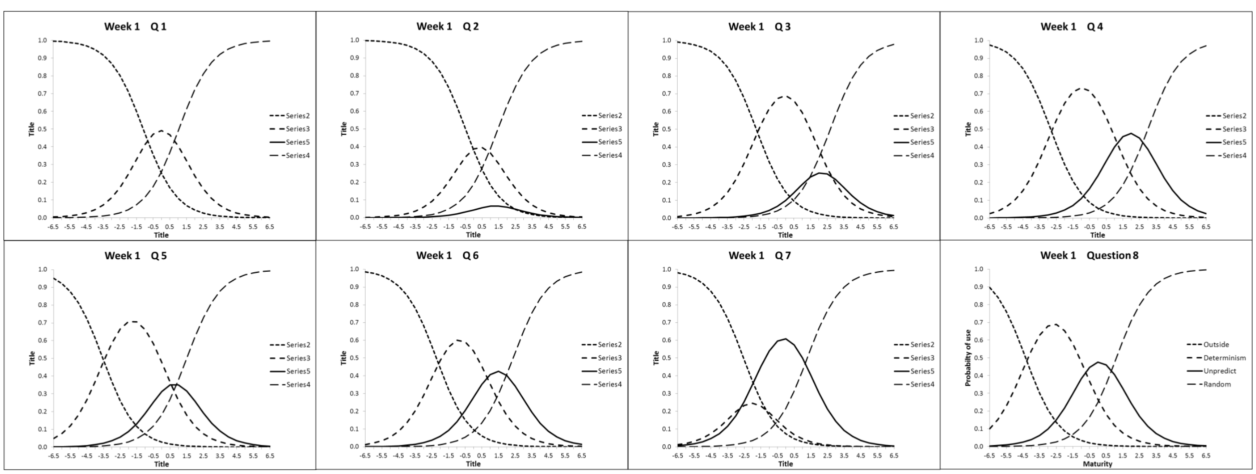

Fig. 6 Item profiles for each topic/question set. On the x-axis is the latent ability/ advancedness scale, as explained earlier.

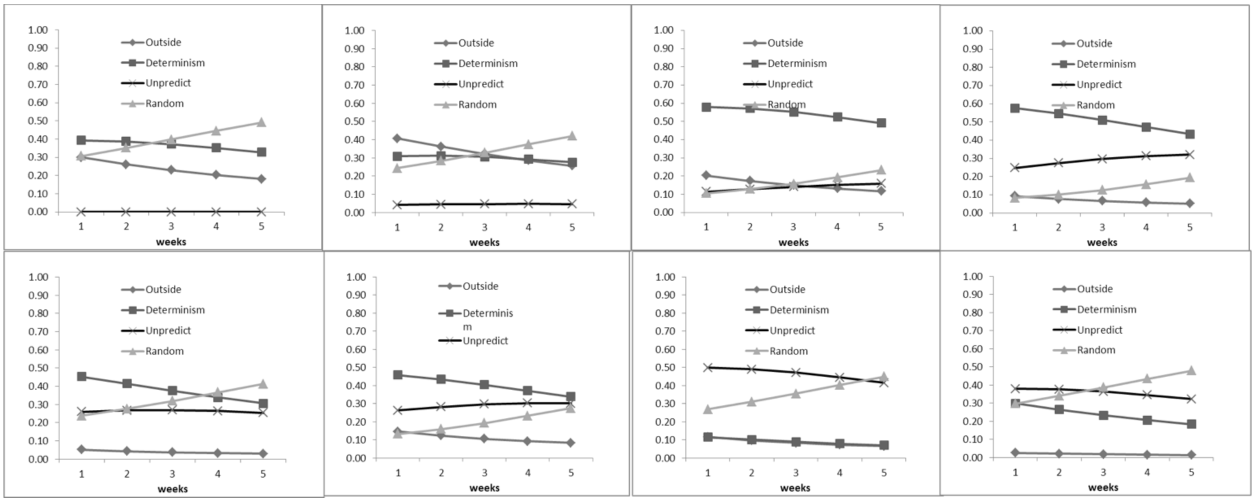

Fig. 7 Model Imlied (expected) week trends of use of categories (levels of understanding).

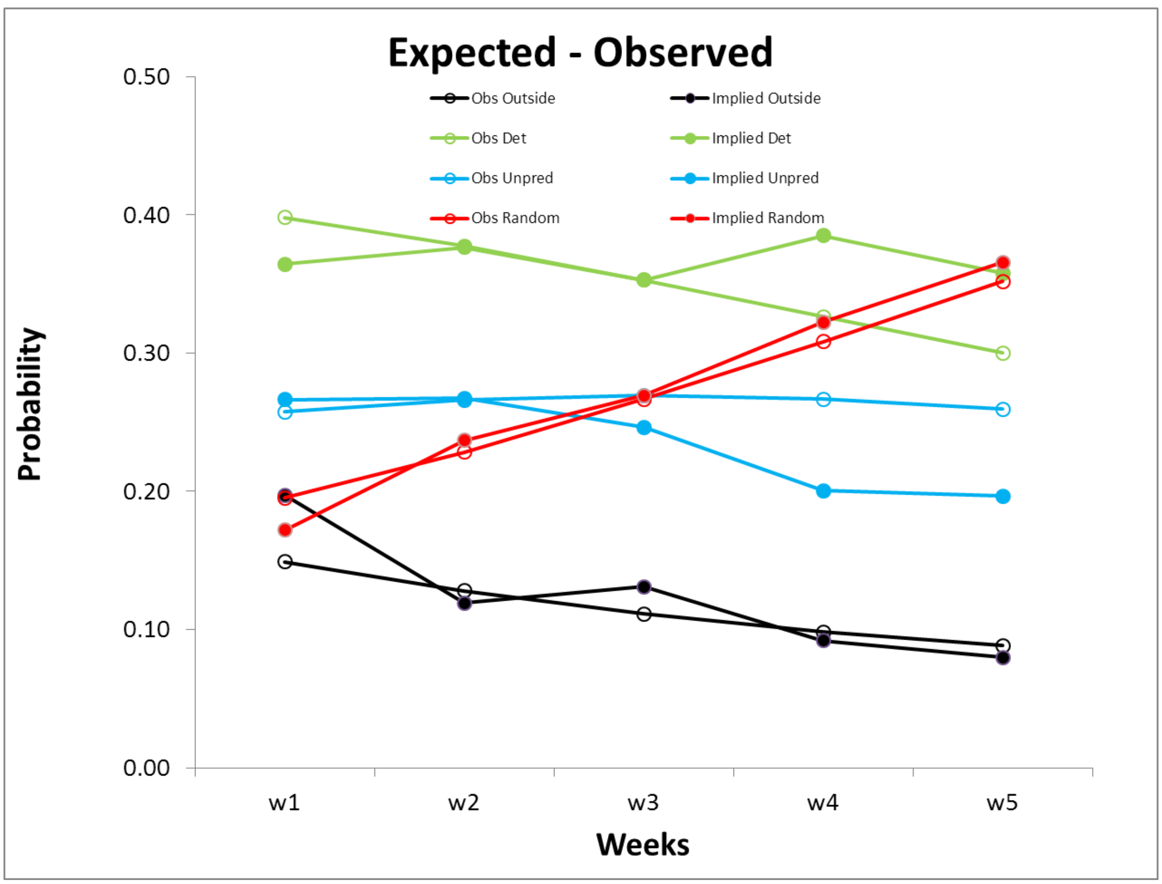

Fig. 8 Observed and model implied (is expected) age trends averaged over all 8 topics/questions. Probabilities refer to using one of the four possible categories (reflecting progressive levels of understanding).