Hierarchical Complexity in Physics

Kristian Stålne
Lund University
Michael Lamport Commons
Harvard Medical School
Eva Yujia Li
Harvard Graduate School of Education

Author Note

Correspondence regarding this paper should be addressed to Dr. Kristian Stalne, Department of Construction sciences, Lund University. E-mail: kristian.stalne@construction.lth.se

Abstract

The derivation of string theory from the two paradigms of wave theory and of relativity is a stage 14 task. The wave theory may partially be represented by the acoustic wave equation for a fluid in one dimension. The stages of development of the wave equation can be is presented in terms of increasing orders of hierarchical complexity. The derivation, shown from order 8 concrete to 13 paradigmatic is presented as a schema where it is specified how a higher order is created by coordinating elements from the respective previous order. The wave equation at the paradigmatic order is created by coordinating the three metasystematic relationships: Newton's Law of Motion, the Constitutive equation and the Ideal gas law. These three relationships in turn coordinate the variables force, density and acceleration, all being systematic since they are functions of time and location. This result gives an understanding of how knowledge is organized in the acoustic domain and in adjacent domains such as classical and solid mechanics. This paradigm is also combined with notions from general relativity to show that the two paradigms may be combined to form a crossparadigmatic task. One result is string theory. It also serves as an illustrative example of the principles of MHC.

Keywords: String theory, wave theory, relativity, Orders of Hierarchical Complexity, crossparadigmatic task

Hierarchical Complexity in Physics

The purpose of this paper is to apply the Model of Hierarchical Complexity (Commons, 2008; Commons, Pekker, 2008; Commons, Trudeau, Stein, Richards, & Krause, 1998) to determine the stage of physics theories. It is an analysis using the Model of Hierarhical Complxity (MHC) of the historical development of String theory and its alternatives and antecendents starting fluid mechanics and their wave equation. The MHC explains why the tasks were ordered as they were and had to unfold in the order they did. But the time, place and person of the development of each advance was not constrained by the MHC and is chaotic. It will show how the wave equation for a fluid in one dimension is derived by increasing orders of hierarchical complexity. It will also be illustrated how the string theory is an Order of Hierarchical Complexity 14 task as a coordination of the laws of quantum mechanics and the theory of relativity. It is important to clarify that the tasks for which the orders of hierarchical complexity will be determined are the tasks of formulating theories. The paper takes perspective of the scientists who formulated the theories. It takes into account what was known at the time the theories were created, and what knowledge the scientists built on to formulate their theories. These facts are revealed by the history of science. This is in contrast with the task of understanding or applying the theories. Once the theories have been formulated, the task of understanding and applying them drops down in stage because of what has been termed support (Fischer, et al, 1984).

The paper first introduces the Model of Hierarchical Complexity and the orders that are derived from it. Then it is demonstrated how the wave equation is being built up through the increasing orders of complexity. The resulting wave equation and its generalization to quantum physics are found to be of the 13th paradigmatic order. String theory, which reconciles quantum physics and the theory of general relativity, is at the 14th cross-paradigmatic order. In order to move up to the 15th order, one has to be able to reflect on actions on the 14th order.

The Model of Hierarchical Complexity

The Model of Hierarchical Complexity is a measurement system that sets forth the hierarchical structure by which actions are put into a hierarchical order. Actions are behavioral events that produce outcomes. A task is a set of required actions to obtain an objective. In the literature, two types of complexities have been identified (Commons, Trudeau, et al, 1998): horizontal (traditional) and vertical (hierarchical). In traditional horizontal complexity, the complexity of a task is determined by the number of times a specific subaction is repeated. In hierarchical complexity, the complexity of an action is determined by the non-arbitrary way in which the subactions are organized, not how many subactions there are (Commons & Pekker, 2008).

In hierarchical complexity, actions at a *higher order of hierarchical complexity* (see figure 1)*:*

a) Are defined in terms of actions at the *next lower* order of hierarchical complexity

b) *Organize* and *transform* the lower-order actions

c) Produce organizations of lower-order actions that are new and *not arbitrary,* and cannot be accomplished by those lower-order actions alone

Once these conditions have been met, we say the higher-order action *co-ordinates* the actions of the next lower order. The task of evaluating $a \times (b + c)$ is used as an example. The standard way to complete this task is to distribute a over b and c is by having $(a \times b) + (a \times c)$. This shows how distribution is built out of the actions of × and +. Contrast this to the case of $(a + b) + c$. Addition is associative and $(a + b) + c$ is equivalent to $(a + b) + c$ or $a + (b + c)$. Therefore, in the task of $(a + b) + c$, the organization of two actions of addition is arbitrary. So evaluating $a \times (b + c)$ is more hierarchically complex than the task of evaluating $(a + b) + c$. The task of distributing is also more hierarchically complex than the two-part task of first evaluating $b + c = d$ and then evaluating $a \times d$.

Actions of each stage coordinate actions that are one stage lower, thus creating a hierarchical system. *Stage of performance* is defined as the highest-order hierarchical complexity of the task solved (Commons, Miller, Goodheart, & Danaher-Gilpin, 2005).

In previous research, tasks have been found to occur at 14 orders of hierarchical complexity, from 0 (calculatory) to 14 (cross-paradigmatic). Table.1 shows the orders of Hierarchical Complexity. Thus far, however, there have been few examples of tasks at the 14 cross-paradigmatic order. The order sequence presumably is infinite, but because of human limitations, we have created only 14 (and possibly 15) orders.

The Wave Equation for a Fluid in One Dimension

The wave equation describes the behavior of waves in a medium. The following equation is the wave equation of a pressure wave in a fluid in one dimension, where p is pressure and c is the speed of sound in the fluid.

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

In the following it will be demonstrated how the wave equation in one dimensional is derived by coordinating more and more complex building blocks, from the 8 concrete order, through each of the following orders, arriving at the final result, the wave equation at the 13 paradigmatic order.

Order 8 Concrete

At the concrete order a particle's or fluid element's state can be given in terms of actual numbers that represent

- Displacement $u_1, u_2, u_3, \ldots$
- Particle velocity $v_1, v_2, v_3, \ldots$
- Particle acceleration $a_1, a_2, a_3, \ldots$
- Pressure $p_1, p_2, p_3, \ldots$
- Density $\rho_1, \rho_2, \rho_3, \ldots$

The respective state can be given at different certain times $t_1, t_2, t_3, \ldots$ and at a certain positions $x_1, x_2, x_3, \ldots$ The subscripts indicate that these variable values are actually specific instances. In logic, they are called specified variable and therefore are concrete.

Order 9 Abstract

Abstract variables are created by coordinating every possible outcome of the concrete instances, specific times or specific positions. The abstract state variables, or field variables as they are referred to, are used that coordinates every possible displacement *u*, velocity *v*, acceleration *a*, pressure *p* and densityρ (rho). At the abstract order, time and position are expressed as variables *t* and *x*.

At the abstract order, the equation for pressure is provided. Even though the equation is from the systematic order, the variable themselves can be viewed as just variables. The equation is given and all a participant has to do is to put in the correct values for the derivatives. The definition provided for change force *dF* per change in unit area *dS* is

$$p = dF/dS$$

The definition provided for density ρ (rho) is change in mass *dm* per change in unit volume *dV*

$$\rho = dm/dV$$

Order 10 Formal

At the formal order, field variables are expressed as single variable functions or deduced by derivation with respect to one variable. The very notion of a function, a mapping relating two variables, input and output, is always formal.

At a fixed location $x = x_0$,

$$
\begin{aligned}
u &= u(t) = \hat{u}\cos(\omega t) \\
v &= v(t) = \hat{v}\cos(\omega t) \\
a &= a(t) = \hat{a}\cos(\omega t) \\
p &= p(t) = \hat{p}\cos(\omega t) \\
\rho &= \rho(t) = \hat{\rho}\cos(\omega t)
\end{aligned}
$$

A letter such as u with the symbol is read "u hat". $\hat{u}, \hat{v}, \hat{a}, \hat{p}$ and $\hat{\rho}$ are the constant amplitudes and ω (omega) is the angular velocity which relates to the period time T according to $\omega = 2\pi/T$.

At a fixed time $t = t_0$ the pressure in one dimension is

$$p = p(x) = \hat{p}\cos(kx)$$

This corresponds to a snapshot of the pressure of a one dimensional travelling wave in a duct, such as an air shaft, as a function of the position x. k is the constant wave number, which can be seen as a spatial angular frequency with is related to the wavelength λ (lambda) according to $k = 2\pi/\lambda$.

Order 11 Systematic

At the systematic order 11, the field variables are expressed as functions of more than one abstract variable. Here the field variables *u, p*, *ρ*, etc are expressed as functions of both time and location, according to,

$$
\begin{aligned}
u(x,t) &= \hat{u}\sin(kx - \omega t) \\
p(x,t) &= \hat{p}\cos(kx - \omega t) \\
\rho(x,t) &= \hat{\rho}\sin(kx - \omega t)
\end{aligned}
$$

Kinematics describes the movement of particles expressed in particle displacement, velocity and acceleration. Velocity and acceleration are defined as the derivatives of displacement and velocity, respectively, with respect to time according to

$$v(x,t) = \frac{\partial}{\partial t}\big(u(x,t)\big)$$
$$a(x,t) = \frac{\partial}{\partial t}\big(v(x,t)\big)$$

The field variables, which are functions at the systematic order, can be grouped into the three categories of kinematics - expressed in displacement, velocity or acceleration, force - expressed in pressure, and mass - expressed in density.

Order 12 Metasystematic

The metasystematic order 12 is characterized by coordination of two or more systems at the systematic order.

Kinetics is achieved by means of Newton's law of motion in rigid body dynamics, which is the coordination of kinematics and force. The derivation of Newton’s law of motion for a fluid is therefore a coordination at the Metasystematic order, since it successfully coordinates the system of force through the pressure $p = p(x,t)$ with the system of kinematics through the acceleration $a = a(x,t)$. Using pressure $p(x,t)$ and particle velocity $v(x,t)$, Newton’s law of motion for a fluid in one dimension can be expressed as,

$$\rho_0 \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial x}$$

where ρ_0 is the mean density of the fluid.

Another example of a metasystematic coordination is the *Continuity Equation*, which is based on the principle of indestructibility of mass. It is a mathematical formulation of the relationship between changes in density $\rho(x,t)$ and changes in volume of an element, which can be expressed with the particle velocity of the element $v(x,t)$, as a function of time and position according to

$$\frac{\partial \rho}{\partial t} = -\rho_0 \frac{\partial v}{\partial x}$$

A third example of a metasystematic coordination is the *Ideal Gas Law*, which gives a relationship between the pressure $p(x,t)$ and the density $\rho(x,t)$. From the ideal gas law the following equation can be derived, where the right hand side only contains constants.

$$\frac{\partial p}{\partial \rho} = \kappa \frac{p_0}{\rho_0}$$

It can be noted that these three examples of relationships at a metasystematic level coordinates the variables that reflects different aspects of the phenomenon, or categories, a wave motion studied as a propagation of force, displacement and mass.

Order 13 Paradigmatic

At the Paradigmatic order, the wave equation is derived by coordinating the three metasystematic relations presented above:

- Newton's law of motion
- The Continuity Equation (Conservation of Mass)
- The Ideal Gas Law

The coordination is performed by employing the three Metasystematic relationships to eliminate two of the field variables, usually velocity and density, to achieve the final result, the wave equation expressed in pressure $p(x,t)$ as a field variable,

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0$$

The solution that satisfies the wave equation describes a travelling wave propagating in a fluid in the positive (first half) and the negative (second half) x-direction.

$$p(x,t) = \hat{p}_+ \cos(kx - \omega t) + \hat{p}_- \cos(kx + \omega t)$$

The same type of relation can be derived for waves in solid media, where shear forces and torques also have to be considered. This will result in not only longitudinal waves but shear, bending, rotational and surface waves as well.

The basic derivation and appearance of the wave equation will be similar for all cases. For example, the classical wave equation, which models a wave on a disturbed string, is expressed by the following, where y is the vertical displacement of a given point at the position x on the string.

$$\frac{\partial^2 y}{\partial t^2} - v^2 \frac{\partial^2 y}{\partial x^2} = 0$$

All the wave equations are at the 13th paradigmatic order.

The wave equation also shows up in quantum mechanics. The time-independent Schrödinger Equation is a decoordination of classical wave equation and the conservation of energy, Total Energy. *TE* = *PE* + *KE*, where *PE* = potential energy, *KE* = kinetic energy. The coordination is at the paradigmatic stage because the conservation of energy is at the Metasystematic stage. The coordination of an action at the paradigmatic stage and an action of metasystematic stage completes a task at the paradigmatic stage. The following equation is a one-dimensional, time-independent Schrödinger Equation for a particle of mass *m*, commonly known as the Time-Independent Schrödinger Equation,

$$-\frac{h^2}{2m} - \frac{d^2y}{dx^2} + V(x)y = Ey$$

Where *m* is the particle mass, y is the vertical displacement of a particle, and *V(x)* is the potential energy of a particle as a function of position *x*.

Order 14 Cross-paradigmatic

At the cross-paradigmatic order, the field of quantum mechanics is reconciled with the theory of general relativity. To understand the reason that this coordination is at the 14th order, it is helpful to review its history and background of the theory of relativity.

Special and General Relativity

Albert Einstein (1950) created a new model of the universe by coordinating the paradigm of the theoretical and experimental result that light travels in a constant speed with the paradigms of classical physics to form the field of relativity.

In the field of electromagnetism, Maxwell's equation gives the result that the speed of light has to be the same to all the time (Toth, 2003). This result conflicts with the laws of classical mechanics. According to Newton's classical mechanics, the speed of a moving object is observed to be different by observers moving at the different speeds. They observe the relative speed of the moving object compared with themselves. The constant speed of light in that theory is paradoxical, because it seems to suggest that the speed of observer does not matter. Maxwell explained this by proposing another theory. He proposed that light has to be transmitted by a type of medium, which he named “ether”, that the universe is full of. Ether is static in the universe. As the earth revolves around the sun, it moves crossing the “ether field”. Maxwell proposed that the speed of light solved by Maxwell’s equation is the “absolute” speed of light in the universe. However, as the earth revolves around the sun, there should exist “relative” speed of light. According to this proposed theory, the speed of light is relative to the speed of the observer.

Michelson and Morley (1887) tested the existence of ether by measuring the speed of light at a static point and at a moving point. Surprisingly, this experiment showed that the speed of light is the same whether or not the observer is moving, disconfirming Maxwell’s theory of ether.

Einstein realized that to accept the speed of light as being constant regardless of the position and speed of the observer is to establish a new space -time model of the universe. He derived the theory of special relativity by keeping the speed of light constant and making time and space flexible. His theory suggested that time and space are contractible. An observer on a fast moving spaceship experiences time slower and space shorter than the observer on a slowly moving spaceship. This theory has been confirmed by experiments, such as showing that the amount of energy goes up as a particle is accelerated towards the speed of light. Einstein created a four-dimensional framework of the universe, three dimensions of space and time. Later, the theory of special relativity was expanded to the theory of general relativity. It made it possible for Einstein to explain gravity and its equivalence to momentum. It also predicted that light would appear to be bent when it passed near the Sun. This is because space time is warped or curved by the mass of the Sun.

The four dimensional space equations are described below. Defining the event to have space-time coordinates (t, x, y, z) in system S and (t', x', y', z') in S', then these coordinates are related in the following way:

$$t' = \gamma(t - \frac{vx}{c^2})$$

$$x' = \gamma\,(\,x - vt\,)$$

$$y' = y$$

$$z' = z$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

is the Lorentz factor, *c* is the speed of light in a vacuum and *v* is the speed of the system *S'* relative to *S*.

The reasons that the coordination is at the pragmatic order 14 is that the relativity theories coordinated two paradigms at order 13. The first paradigm is the model of light waves propagating at a constant speed in vacuum. This is the theoretical conclusion of Maxwell's equations and the experimental result of Michelson and Morley. Understanding the theoretical implication of Maxwell's equation is a task at order 13. Maxwell's equations coordinated two order 12 metasystems, electric field and the magnetic field. Understanding the empirical evidence provided by the experiment was correct is a task at order 11. Einstein abandoned the theory of ether, which marked the end of the early 19th century paradigm of physics in which all waves had to travel in a medium.

The second paradigm is the old paradigm of Newton's laws of mechanics positing a gravitational field within Euclidean geometry and founding mathematical physics. It has distance independent of time and of rate. Understanding the interrelatedness of all of Newton's laws and the properties of the system in classical physics is a task at order 12. These two paradigms had intrinsic conflict with each other concerning the speed of light. Einstein reconciled the two by constructing a new cross-paradigmatic theory in which time, distance, an even mass, are all transformed by showing that all of them are a function of their relative speed with respect to the speed of light. The extension to the general relativity theory, integrates space-time with inertia and gravity.

Because mass in their as in $E = mc^2$, this integrates a new physics, geometry.

String Theory

Quantum mechanics describes the properties of particles at the subatomic level. It describes the subatomic particles as operating with uncertainty and probability, according to the Heisenberg uncertainty principle. Quantum mechanics successfully explains three of the four fundamental forces in physics, the strong force, the weak force and the electromagnetic force. According to quantum mechanics, forces are created by the exchange of messenger particles. For example, electromagnetic force is created by the exchange of photons. The more exchange, the stronger the forces. However, this theory does not explain the last fundamental force of physics, gravity ("Sting Theory", 2011).

Einstein's ("Sting Theory in Two Minutes", 2011) theory of general relativity describes the gravity as a function of space and time. The theory of general relativity is an improvement over Newton's immutable mechanics. It describes the universe as mechanical and predictable. This theory can be observed with massive objects, but not with the microscopic particles, characterized by chaotic movements and unpredictability.

Both theories have been experimentally tested and proven valid. However, general relativity and quantum mechanics seem to be incompatible with each other. They paint distinct pictures of the universe – one operates under mechanical laws and the other filled with uncertainty. They also have disjointed experimental domains. General relativity is only observable with massive objects. Quantum effects are only observable with minute particles. Could there be a single unified theory that explains the universe on both the macro and on the micro scale?

No theory to date has successfully reconciled quantum mechanics and general relativity. However, there are a few plausible working models. String theory is an active research framework in the field of physics. It proposes that everything in the universe is composed of tiny vibrating strings. The shape of the string and the way that the strings vibrate contribute to matters' unique properties, such as mass. The string theory describes that gravity is produced by one type of vibrating string called the graviton. It offers an explanation of how gravity works in the subatomic scale. This is the key to unifying the four forces, gravity, the strong force, the weak force, and electromagnetic force.

There are also other alternatives to string theory that unite the two camps. Examples are Loop Quantum Gravity and Quantum Gravity.

These theories are at the 14th cross-paradigmatic order because they successfully coordinate two theories at the paradigmatic order.

Order 15

At order 15, the action required in the transition is to reflect on Order 14 tasks. Scoring order 14 tasks is, but not completely, an order 15 task. It is in transition because one has to be at a higher stage in order to score the lower stages. An order 15 tasks requires a reflection on a stage 14 task and what is missing from it. The reason that it is transitional is that there is not a positive description of the order and how it coordinates two or more cross-paradigmatic order tasks. The order sequence presumably is infinite, but because of human limitations, we have created only 14 and possibly 15 Orders.

Conclusion

The derivation of the wave equation for a fluid in one dimension serves as an illustrative example of the principles of the Model of Hierarchical Complexity. Lower stage elements are coordinated by higher stage systems, and the systems become increasingly more complex hierarchically. It is shown that the resultant wave equation is at the 13^{th} paradigmatic order. The wave equation is generalized to describe wave properties of particles in the quantum realm. At the 14^{th} cross-paradigmatic order, quantum physics and the theory of general relativity are reconciled by the string theory. At the next order above, the ability to reflect on a 14^{th} order task is attained. This result gives an understanding of how knowledge at the highest known orders of human performance is organized. This result can be generalized to other domains and support progress in areas that have not yet reached that high in complexity.

References

Commons, M. L. (2008). Introduction to the model of hierarchical complexity and it relationship to postformal action. *World Futures: Journal of General Evolution, 65*(1-3), 305-322.

Commons, M. L., Bresette, L. M., & Ross, S. N. (2008). The connection between postformal thought and major scientific innovations. *World Futures: Journal of General Evolution, 65*(1-3), 503-512.

Commons, M. L., & Pekker, A. (2008). Presenting the formal theory of hierarchical complexity. *World Futures: Journal of General Evolution, 65*(1-3), 375-382.

Commons, M. L., Trudeau, E. J., Stein, S. A., Richards, F. A., & Krause, S. R. (1998). The existence of developmental stages as shown by the hierarchical complexity of tasks. *Developmental Review, 8*(3), 237-278.

Einstein, A. (1905). On a heuristic viewpoint concerning the production and transformation of light. *Annalen der Physik 17*(6): 132–148. doi:10.1002/andp.19053220607

Michelson, A. & Morley, E. (1887), "On the Relative Motion of the Earth and the Luminiferous Ether". *American Journal of Science, 34*, 333–345.

Skudrzyk, E. (1971). *The Foundations of Acoustics – Basic Mathematics and Basic Acoustics.* Verlag, Wien, Austria: Springer

String Theory (2 Minutes) (2009). Available from http://www.youtube.com/watch?v=_B0Kaf7xYMk

String Theory (2011). Available from http://www.youtube.com/watch?v=E7FV9aaiwKQ

Toth, V. (2003). Why is the speed of light constant? Retrived from http://www.vttoth.com/LIGHT/light.htm

Table1

Orders of Hierarchical Complexity

Order	Name Complexity	7	Primary
0	Calculatory	8	Concrete
1	Sensory & Motor	9	Abstract
2	Circular Sensory-motor	10	Formal
3	Sensory-motor	11	Systematic
4	Nominal	12	Metasystematic
5	Sentential	13	Paradigmatic
6	Preoperational	14	Cross-paradigmatic

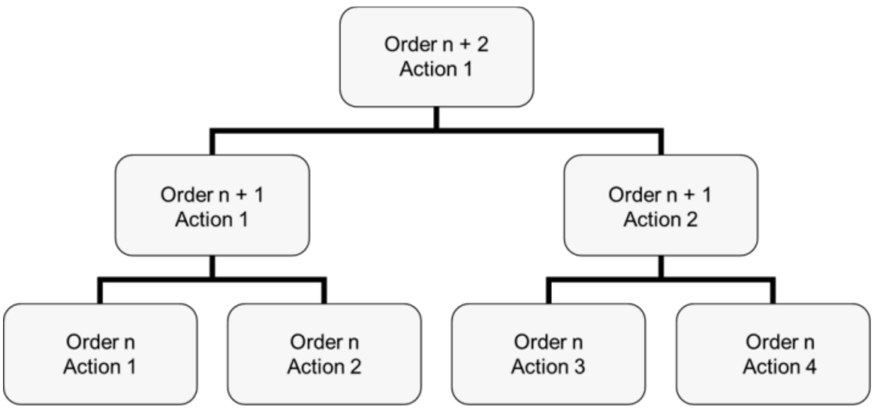

Fig. 1 Hierarchical Structure of Tasks